Analyzing Frame Conditions in UML/OCL Models:  
Consistency, Equivalence, and Independence

Philipp Niemann¹, Nils Przigoda², Robert Wille¹,³ and Rolf Drechsler¹,⁴  
¹Cyber-Physical Systems, DFKI GmbH, Bremen, Germany  
²Siemens AG, Braunschweig, Germany  
³Institute for Integrated Circuits, Johannes Kepler University Linz, Linz, Austria  
⁴Group for Computer Architecture, University of Bremen, Bremen, Germany  
philipp.niemann@dfki.de, nils.przigoda@siemens.com, robert.wille@jku.at, drechsle@informatik.uni-bremen.de

Keywords: UML/OCL, Operation Contracts, Frame Conditions, Validation and Verification

Abstract: In behavioral modeling using UML/OCL, operation contracts defined by pre- and postconditions describe the effects on model elements (such as attributes, links, etc.) that are enforced by an operation. However, it is usually not clearly stated which model elements can be affected and which shall not, although this information is essential in order to obtain a comprehensive description. A promising solution to this so-called frame problem is to define additional frame conditions. However, properly defining frame conditions which complete the model description in the intended way is a non-trivial, tedious and error-prone task. While for UML/OCL models in general, methods for validation and verification are available, no analysis methods for frame conditions exist so far that could support the designer in this process. In this work, we close this gap and propose a set of primary analysis objectives (namely consistency, equivalence, and independence) that provide substantial information about the correctness and adequateness of given frame conditions. Moreover, we formalize these objectives as to be able to conduct the corresponding analyses in an automatic fashion using the deductive power of established approaches for model validation and verification. Finally, we discuss how the resulting methodology can actually be applied and demonstrate its potential for elaborated analyses of frame conditions.

1 Introduction

The design of software as well as hardware systems has become an increasingly complex task. The introduction of modeling languages aims to aid designers in this process by providing description means that abstract from implementation details but remain precise enough to specifically describe the intended system. Nowadays, the Unified Modeling Language (UML) (Rumbaugh et al., 1999) is one of the standard modeling languages which allows, e.g., the description of a design by means of class diagrams. Since UML version 1.1, the respective models can additionally be enriched by descriptions formulated in the Object Constraint Language (OCL) (OMG – Object Management Group, 2014)—a declarative language that allows to impose additional textual constraints which further refine properties and relations between the respective model elements (such as attributes, links, etc.). Overall, this allows to define valid system states by invariants and to describe the behavior of operations by means of pre- and postconditions—eventually yielding UML/OCL models that precisely describe the structure and behavior of the system.

A well-known shortcoming of the resulting declarative descriptions is that pre- and postconditions often do not make clear enough what may or may not be modified in a transition between two system states. In fact, they only define restrictions of the calling and the succeeding system state, respectively, but do not specify precisely what is within the frame that might be modified by an operation—possibly allowing for unintended behaviour. This so-called frame problem (Borgida et al., 1995) does not only occur in UML/OCL, but also in many other languages that use declarative descriptions like, e.g., Eiffel, Z, JML, VDM, or CML. Consequently, there has been a large body of research on this problem. A common approach to cope with it and avoid unintended behaviour is to provide additional constraints in terms of so-called frame conditions. While each of the mentioned languages has built-in functionalities for this
purpose, mechanisms for specifying frame conditions in UML/OCL have been suggested only recently (Ko- 
siučzenko, 2013; Brucker et al., 2014).

However, while frame conditions are indeed able to solve the frame problem, properly defining them is a non-trivial process. Similarly as the definition of the UML/OCL model itself, it requires a full understanding of the considered system as well as its dependencies. But while the designer is aided by several tools and methods when defining the UML/OCL model (see, e.g., (Gogolla et al., 2007; Gogolla et al., 2009; Denmuth and Wilke, 2009)), almost no support exists yet for the proper definition of frame conditions. In fact, initial approaches providing the designer with proposals for frame conditions and/or a classification of model elements that may be affected by an operation have recently been proposed in (Niemann et al., 2015). But they cannot guarantee that the derived frame conditions are indeed correct or complete the specification of the model in the actually intended way. While for UML/OCL models in general, corresponding methods for validation and verification are available (see, e.g., (Anastasakis et al., 2007; Cabot et al., 2008; Cabot et al., 2009; Brucker and Wolff, 2008; Choppy et al., 2011; Soeken et al., 2011; Hilken et al., 2014; Przigoda et al., 2015a; Przigoda et al., 2016)), no dedicated analysis method for frame conditions exists so far.

In this work, we close this gap by providing a methodology for the dedicated analysis of frame conditions in UML/OCL models. To this end, we first discuss primary objectives for such an analysis—yielding a notion of consistency, equivalence, and independence of frame conditions. Based on that, a method is introduced afterwards, which automatically analyzes a given set (or sets) of frame conditions with respect to these objectives. An application of the resulting methodology confirms the benefits of the proposed approach. In fact, designers are aided with a tool that allows them to efficiently check whether the derived frame conditions are consistent with the given UML/OCL model and complete the specification of the model in the actually intended way.

The remainder of this paper is structured as follows: All ideas and concepts covered in this work are illustrated by means of a simple UML/OCL model specifying an access control system which serves as a running example and is introduced in Section 2. Afterwards, Section 3 briefly reviews the frame problem as well as the different UML/OCL description means introduced in the past to define frame conditions. Based on that, primary objectives for analyzing the respectively obtained frame conditions are introduced and discussed in Section 4 and an automatic method for conducting these analyses is described in Section 5. Finally, an implementation of the resulting methodology is discussed in Section 6 and the paper is concluded in Section 7.

2 Preliminaries

In this section, we introduce basic concepts and notions of UML/OCL by means of the running example that will also be later on used to illustrate the basic concepts of frame conditions as well as the proposed analysis methodology.

The running example, a slightly modified version of the one originally presented in (Przigoda et al., 2015b), specifies a control system which grants access to buildings based on magnetic cards as authentication method. The cards are checked at turnstiles at the buildings’ entries and exits. The system model is given in terms of a UML class diagram enriched with textual OCL constraints and is depicted in Fig. 1. The pure UML part describes the structure of the system in terms of classes (e.g., Building, MagneticCard, Turnstile), attributes and available operations of each class (e.g., Building::inside or Turnstile::goThrough()) as well as relationships between the classes in terms of associations. For the sake of a convenient reference, we will refer to the union of all attributes (of all classes) together with all relations of a model as the set of model elements.

In this particular case, there is a single relationship stating that each turnstile is associated with a unique building and that each building contains at least two turnstiles (gates). Such multiplicity constraints—besides inheritance of classes which is not present in the running example—are essentially the only constraints that can be stated in class diagrams using pure UML.

To enforce further constraints or properties of a system, textual OCL constraints are applied. On the one hand, invariants describe properties such as the uniqueness of a magnetic card’s ID (invariant uniqueID), the existence of at least one entry and one exit for each building (invariants atLeastOneEntry and atLeastOneExit) or the fact that permanently either the green or the red light of a turnstile is lit (invariant eitherGreenOrRedLight). On the other hand, OCL is employed to formulate so-called operation contracts (Meyer, 1992) which comprise preconditions (denoted by ≪) that are necessary to invoke an operation call in the first place as well as postconditions (denoted by ≫) that can be taken for granted after the execution of the operation has been completed. For instance, the operation
checkCard() can only be invoked on a turnstile if its green light is not on (precondition). The state of the green light after the operation has been executed depends on whether (a) the inserted card is in principle authorized to enter/leave the building (building.authorized->includes(card.id)) and whether (b) the card has been inserted on the “expected” side of the turnstile (entry <> building.inside->includes(card.id); the second part has been added in order to prevent multiple persons from using the same card to enter/leave a building one after the other). If these checks are passed, the postconditions enforce that the green light is lit and the ID of the inserted card is stored in the attribute currentlyAuthorized.

All these constraints determine which instantiations of the model (system states) and operation calls (transitions) are valid and which are not:

- A system state σ is a set of objects together with attribute values (instantiations of classes) and interconnected links (instantiations of associations). A state σ is termed valid if, and only if, it satisfies all UML constraints (multiplicity and inheritance) as well as all OCL invariants.

- A transition between two system states σ₁, σ₂ through an operation call ω (i.e., an operation oP₁ called on some object from σ₁) is termed valid if, and only if, the preconditions ω₁ of ω are satisfied in σ₁ and the associated postconditions ω₂ are satisfied in σ₂.¹

A valid transition is denoted as σ₁ ⊢ω σ₂ and is termed valid execution scenario if, and only if, also both system states σ₁ and σ₂ are valid (which is not required in the definition of valid transitions).

¹Note that the postconditions might also refer to the pre-state of the operation (σ₁) using the suffix @pre as, e.g., for Turnstile::goThrough() in the running example.

3 Frame Problem and Frame Conditions

This section briefly reviews the frame problem of behavioral models and presents the state-of-the-art for the specification of frame conditions which are employed to address this problem. Based on that, the resulting validation gap and the proposed analysis methodology are discussed in detail in Section 4.

In UML/OCL class diagrams, behavior is expressed in terms of operations with pre- and postconditions. At first glance, these declarative descriptions of the operation’s behavior ideally fit to the paradigm of designing systems without the need to provide detailed implementations. However, a closer look re-
veals that this may allow for undesired behavior.

**Example 2.** Consider again the valid execution scenario shown in Fig. 2. Recall that both system states are valid, i.e. all model constraints are satisfied. Moreover, also the transition from the system state on the top to the one on the bottom is valid, since for the operation checkCard(M1) (called on turnstile T1) the corresponding preconditions (postconditions) are satisfied in the top (bottom) system state.

More precisely, as intended by the designer, the green light of turnstile T1 is turned on and the ID of M1 is stored in the currentlyAuthorized attribute. However, at the same time it is also possible to turn on the green light of the other turnstile T2 or to add an arbitrary ID (e.g., 42) to the inside attribute of the building B1 as highlighted in red and italics in Fig. 2. Although such a behavior is obviously not intended, it is completely in line with the postconditions.

In general, the shortcoming of declarative descriptions like pre- and postconditions is that they often do not make clear enough which model elements are allowed to change during an operation call. In other words, they do not specify what is within the frame that might be modified by an operation—the so-called frame problem (Borgida et al., 1995). As a consequence, the resulting model/description is underspecified and additional frame conditions need to be formulated.

To this end, note that the frame problem also arises in the context of software verification where a substantial body of research has focused on possible solutions (see, e.g., (Beckert and Schmitt, 2003)) and corresponding approaches have been integrated into several verification tools like Boogie (Leino, 2008) or KeY (Ahrendt et al., 2005). Unfortunately, these approaches are not directly applicable to UML/OCL for various reasons, especially due to the fact that (a) OCL principally allows one to access arbitrary objects via allInstances(), (b) associations are always bi-directional (in contrast to uni-directional pointers) such that changes to references always affect both ends, and (c) object creation and deletion can be rather random as we usually do not have a precise implementation.

Nonetheless, recently there have been several dedicated proposals for the specification of frame conditions in UML/OCL models which are inspired by the above approaches. More precisely, the following approaches have been suggested:

- **Explicit Postconditions:** A straightforward approach is to explicitly specify what is not in the frame by extending the postconditions with constraints like modelElem = modelElem@pre.

```ocl
context Turnstile::checkCard(card : MagneticCard): ...
-- FrameConditions for Class Turnstile
post : Turnstile.allInstances@pre --> forAll | t | t.entry = t.entry@pre
    and t.building = t.building@pre
    and (self <> t) implies
    { t.greenLightOn = t.greenLightOn@pre
        and t.redLightOn = t.redLightOn@pre
        and t.currentlyAuthorized = t.currentlyAuthorized@pre
    }
    ...

post : Turnstile.allInstances@pre() = Turnstile.allInstances()
-- FrameConditions for Class Building
post : Building.allInstances@pre() = Building.allInstances()
-- FrameConditions for Class MagneticCard
post : MagneticCard.allInstances@pre() = MagneticCard.allInstances()
```

Figure 3: Frame conditions for checkCard(…) using the explicit postconditions approach

The corresponding conditions for the operation Turnstile::checkCard(…) from the running example are listed in Fig. 3. This listing, but even more the case study in (de Dios et al., 2014), illustrates very impressively the drawback of this approach: it is time-consuming to manually create the constraints in the first place and to maintain them later on in the case of design changes.

- **Modifies only statements:** A complementary approach has been suggested by Kosiuczenko (Kosiuczenko, 2006; Kosiuczenko, 2013). The idea is to specify the set of variable model elements, i.e., model elements that are allowed to be changed during an operation call, at the same level as pre- and postconditions in terms of modifies only statements. These are of the form

  modifies only: scope::modelElement.

---

2*Modifies only statements were originally introduced as invariability clauses by Kosiuczenko (Kosiuczenko, 2006). A variation of this idea is to specify the set of variable model elements within the postconditions using an OCL primitive modifiedOnly(Set) (Brucker et al., 2014).*
Figure 4: Frame conditions for checkCard(...) and goThrough() using the modifies only approach

Figure 5: List of model elements referenced within the postconditions of checkCard(...)

For instance, the clause modifies only: self::greenLightOn expresses that the operation may only change the attribute greenLightOn of the turnstile on which the operation is called (self). Likewise, the complete frame conditions for the operations Turnstile::checkCard(...) and Turnstile::goThrough() are shown in Fig. 4. Note that the scope can also be more complex than just self and may contain navigation or collections as in Line 13. In addition, it is even possible to allow objects of a certain class to be created or deleted during an operation call using the construct Class::allInstances().

This approach enables the designer to precisely define frame conditions in a much more comfortable, understandable, and maintainable fashion. Moreover, there exists a methodology to assist the designer in the initial generation of the frame conditions (Niemann et al., 2015a) and an approach that does most of the work automatically and requests feedback of the designer in ambiguous cases only (Niemann et al., 2015b).

- Nothing Else Changes: Another approach to the specification of frame conditions is to not write them down explicitly, but automatically derive them from the postconditions using a paradigm such as nothing else changes (Cabot, 2006; Cabot, 2007). Following this paradigm, every model element that is referenced within the postconditions is included in the frame of what may change (and nothing else). In the best case, this implicit approach requires no additional efforts by the designer. However, in general, the resulting frame conditions are often not exactly what the designer intended and it can be non-trivial to adjust them manually—which would have to be done by rewriting the postconditions or adding further ones. For instance, Fig. 5 lists all model elements which are referenced within the postconditions of the operation Turnstile::checkCard(...) from the running example. Only the very first and very last of them, i.e., self.greenLightOn and self.currentlyAuthorized, are actually meant to be affected. In addition, both self.greenLightOn and also self.redLightOn have to be variable in order to fulfill the invariant eitherGreenOrRedLight. To make this implicit dependency transparent to the automatic derivation approach, the particular invariant is added as another postcondition as shown in Fig. 6 (Lines 3–4). To fix the values of the other elements, postconditions as listed in the remainder of Fig. 6 have to be added. Note that, as it is not clear which instance of MagneticCard is used for the card parameter, the id attributes of all MagneticCards are marked as variable by the approach and, hence, have to be restricted manually. Moreover, the additional postconditions contain calls to Class.allInstances() (Lines 9 and 16) which again would be interpreted as referenced model elements and allow for the creation and deletion of objects (of Class). To avoid this, further postconditions have to be added (Lines 13 and 19).

Overall, frame conditions are very important for obtaining complete model descriptions and are a key ingredient when considering the behavior of UML/OCL models. Various approaches to their specification exist, each with complementary strengths and weaknesses.

4 Analysis of Frame Conditions

While using frame conditions as reviewed above indeed solves the frame problem, properly defining them remains a non-trivial process. To this end, the designer needs to fully understand the considered model as well as its dependencies. While initial approaches such as the one proposed in (Niemann et al., 2015b) may aid him or her in this process, they cannot guarantee that the derived frame conditions are indeed correct or complete the specification of the model in
In this section, the three above-mentioned analysis objectives are illustrated in more detail and described in a formal way in order to allow for an automatic analysis (which will be discussed in the following section). To this end, we study the impact of frame conditions on the set of valid execution scenarios (cf. Section 2). Recall that this set (in the following denoted by $S$) is constituted by all valid transitions $\sigma_1 \xrightarrow{\omega} \sigma_2$ between valid system states $\sigma_1, \sigma_2$. A transition is induced by an operation call $\omega$ which consists of an object $o_\omega \in \sigma_1$, an operation $op_\omega$ that is called on it and a (possibly empty) set of parameters. The transition $\sigma_1 \xrightarrow{\omega} \sigma_2$ is termed valid if, and only if, the preconditions $\sigma_1$ of $\omega$ are satisfied in $\sigma_1$ and its postconditions $\sigma_2$ are satisfied in $\sigma_2$ (see the top of Fig. 7). Note that the particular operation $op_\omega$ can be arbitrary for the transitions in $S$.

Now, to focus on individual operations, we classify the valid execution scenarios of a model by the corresponding operation $op_\omega$. This yields a partition of the set of all valid scenarios of a model into disjoint subsets $S_{op} = \{ \sigma_1 \xrightarrow{\omega} \sigma_2 \in S \mid op_\omega = op \}$ for each operation $op$ (of any class) of the model (see the center of Fig. 7). Note, however, that only pre- and post-conditions, but no frame conditions have been taken into account so far. Consequently, in order to analyze a particular set of frame conditions, we further restrict to those execution scenarios that additionally satisfy the given frame conditions (denoted by $f$) and consider the corresponding subsets $S_{op}^f \subset S_{op}$ (see the bottom of Fig. 7).

Using this notation, the analysis objectives mentioned above can be formalized as follows:

### 4.1 Consistency

The major criterion for the quality and validity of well-defined frame conditions is that they are consistent with the (original) contractual specification of the operation. More precisely, assuming that an operation contract in terms of pre- and postconditions is free of contradictions and in principle allows for an execution of the operation ($S_{op} \neq \emptyset$), this property shall be preserved when additionally enforcing the frame conditions ($S_{op}^f \neq \emptyset$). In other words, frame conditions can only be considered consistent, if they are compatible with at least one execution scenario.

To strengthen the significance of this objective, the same compatibility can be required for a set of pivot scenarios $P \subset S_{op}$ (provided by the designer) that characterize the intended behavior of the oper-
ation, i.e., we check whether $P \subset S_{\text{op}}$. In a similar fashion, one may also employ scenarios characterizing unintended behavior and, thus, being incompatible to well-defined frame conditions. For most significant results, the pivot scenarios shall cover the operation’s functionality as comprehensively as possible, i.e., affect as many model elements as possible, be as complementary as possible, and desirably also cover corner-cases.

**Example 3.** Consider the operation `checkCard()` from the running example (Fig. 1) together with the frame conditions specified in Fig. 3. The operation contract in principle allows for an execution of the operation ($S_{\text{checkCard}}(\cdot) \neq \emptyset$), since the transition from Fig. 2 (termed $\omega_0$ in the following) is a valid execution scenario as shown above. However, the frame conditions do not allow the changes highlighted in red: $B_1::\text{inside}$ is required to remain constant in Line 21 of Fig. 3 and switching the lights of $T_2$ is prohibited by Lines 8–11. Overall, this means $\omega_0 \notin S_{\text{checkCard}}$. Nonetheless, the frame conditions themselves are clearly consistent. For instance, when refraining from the changes highlighted in red, i.e., the attributes of $B_1$ and $T_2$ do not change, the resulting transition (shown in Fig. 8(a)) is still a valid execution scenario and is also compatible with the frame conditions. A meaningful set of complementary pivot scenarios would cover the cases of leaving and entering the building (cf. Figs. 8(a) and 8(b)) as well as checking an authorized or unauthorized card (Fig. 8(c)).

### 4.2 Equivalence

Aiming at the relationship between different sets of frame conditions, the first important objective is to check for equivalence. More precisely, given two sets of frame conditions $F_1$ and $F_2$ we are interested to know whether they lead to the same set of valid execution scenarios ($S_{\text{op}}^{F_1} = S_{\text{op}}^{F_2}$) or, if not, what the reasons for the non-equivalence are. To this end, we aim to find scenarios that are only compatible with one set of frame conditions, but not with the other, i.e., scenarios from the set $S_{\text{op}}^{F_1} \Delta S_{\text{op}}^{F_2} = (S_{\text{op}}^{F_1} \setminus S_{\text{op}}^{F_2}) \cup (S_{\text{op}}^{F_2} \setminus S_{\text{op}}^{F_1})$ (symmetric difference). The check can be performed on sets of frame conditions that are only slight variations of each other, but also if they are specified using different approaches/formalisms. Again, pivot scenarios can be employed to prove equivalence on a relevant subset of scenarios or to allow for a more detailed analysis of the differences.

**Example 4.** Consider again the operation `checkCard()` from the running example (as in the previous example). Comparing the frame conditions from Fig. 3 (specified as explicit postconditions) and Fig. 4 (specified using modifies only statements) shows that they are indeed equivalent. However, if the second modifies only statement regarding `self::redLightOn` would have been forgotten in the specification, an evaluation of the pivot scenarios from Fig. 8 shows that only the third one is still compatible, while the first two scenarios are no longer compatible. A deeper analysis of the second scenario reveals that only $T_2::greenLightOn$ and $T_2::redLightOn$ are modified which provides a hint on the missing modifies only statement for `self::redLightOn`.

### 4.3 Independence

The second objective aiming at analyzing the relationship between different sets of frame condition addresses dependencies between individual frame conditions. To this end, two sets of frame conditions $F_1, F_2$ are combined to a set $F_1 \cup F_2$ whose frame of change is essentially the union of the respective frames of $F_1$ and $F_2$. In other words, a model element is allowed to be modified according to $F_1 \cup F_2$.

Figure 8: Pivot scenarios for the operation `checkCard()`.
if, and only if, it is allowed to be modified according to at least one set of frame conditions.

Then, several different cases are possible:

- \( F_1 \cup F_2 \) is consistent (\( S^{F_1 \cup F_2}_{op} \neq \emptyset \)), although neither \( F_1 \) nor \( F_2 \) (considered separately) are consistent. This means that \( F_1 \) and \( F_2 \) require each other.

- \( F_1 \cup F_2 \) and \( F_i \) are consistent \((i = 1 \text{ and/or } i = 2)\). This means that \( F_i \) is independent from the other set of frame conditions \( F_{3-i} \).

- \( F_1 \cup F_2 \) is not consistent (\( S^{F_1 \cup F_2}_{op} = \emptyset \)), although \( F_1 \) or \( F_2 \) (considered separately) are consistent. This means that \( F_1 \) and \( F_2 \) exclude each other.

- Neither \( F_1 \cup F_2 \), nor \( F_1 \), nor \( F_2 \) are consistent.

This only implies that \( F_1 \) and \( F_2 \) are not sufficient to obtain complete or consistent frame conditions.

In order to obtain more detailed information about the particular dependencies, we can go down to the level of model elements and analyze what happens if particular model elements are not only allowed to be modified, but are required to actually be subject to changes. More precisely, we consider a set of model elements \( M = \{m_1, \ldots, m_k\} \) (all included in the frame of \( F_1 \)) together with another model element \( m_0 \notin M \) (included in the frame of \( F_2 \)). If all elements from \( M \) are actually changed in an execution scenario, \( m_0 \) can either (a) be forced to be modified as well, (b) be forced to remain constant, or (c) be allowed to behave either way, i.e., do not have an immediate dependency to the model elements in \( M \).

**Example 5.** Consider the operation \( \text{goThrough()} \) from the running example together with the modifies only statements as listed in Fig. 4. Set \( F_1 \) to be the first modifies only statement \( \text{self::greenLightOn} \) and \( F_2 \) to contain the two remaining statements \( \text{self::redLightOn and self.building::inside} \). Then, \( F_2 \) requires \( F_1 \) and vice versa. In fact, it can be shown that a modification of \( \text{building.inside} \) and/or \( \text{self.redLightOn} \) implies that also \( \text{self.greenLightOn} \) needs to be modified. On the contrary, a change to \( \text{self.greenLightOn} \) also requires \( \text{self.redLightOn} \) to be modified, but not necessarily \( \text{building.inside} \). In fact, if the ID stored in \( \text{currentlyAuthorized} \) is—by incidence—logically already inside/outside the building, a building can be entered/left with no change to \( \text{building.inside} \) (cf. Fig. 9). This dependency becomes apparent at the level of frame conditions if one moves the second modifies only statement \( \text{self::redLightOn} \) from \( F_2 \) to \( F_1 \). Then, \( F_2 \) still requires \( F_1 \), but not vice versa.
5 Automatic Analysis of Frame Conditions

In order to automatically analyze the objectives introduced above, we propose to employ approaches for automatic reasoning on UML/OCL models. To this end, we first review corresponding approaches in Section 5.1. Afterwards, we describe in Section 5.2 how the respective objectives can be formulated on top of these solutions.

5.1 Automatic Reasoning on UML/OCL

In the recent past, several approaches for automatic reasoning on UML/OCL models have been proposed which aim at the validation and verification of structural as well as behavioral aspects (see, e.g., (Anastasakis et al., 2007; Cabot et al., 2008; Cabot et al., 2009; Brucker and Wolff, 2008; Choppy et al., 2011; Soeken et al., 2011; Hilken et al., 2014; Przigoda et al., 2015a; Przigoda et al., 2016b)). Here, we focus on approaches using solvers for problems of Boolean Satisfiability (SAT) or Satisfiability Modulo Theories (SMT), see, e.g., (Hilken et al., 2014; Przigoda et al., 2016b).

The general idea of these approaches is sketched by means of Fig. 11: Instead of explicitly enumerating all possible system states and operation calls, they utilize a symbolic formulation of the given UML/OCL model which allows to consider all possible sequences of system states and operation calls at the same time (up to a given sequence length n).\(^3\)

For this purpose, the model is expressed as a set of variables which can describe arbitrary system states \(\sigma_1, \ldots, \sigma_n\), i.e., the instantiated objects, their attributes and associations, as well as arbitrary transitions—each of which is triggered by a (single) operation call \(\omega_1, \ldots, \omega_{n-1}\). Note that this formulation in principle also covers invalid system states as well as invalid transitions. Consequently, in order to restrict to valid states only, additional constraints over these variables are applied to enforce the model’s static constraints such as multiplicity constraints and OCL invariants. Analogously, in order to ensure valid transitions, pre-, post-, and frame conditions of each possible operation call are also translated to constraints over the state variables, but are only enforced if the transition \(\omega_i\) is chosen to be the corresponding operation. More precisely, the following formulation is applied:\(^4\)

Formulation 1. For a sequence of system states \(\sigma_1, \ldots, \sigma_n\), let \(\Omega_i\) be the set of all operation calls that are available within system state \(\sigma_i\) (\(i = 1, \ldots, n\)). Then, for each of the transitions \(\omega_i\) (\(i = 1, \ldots, n - 1\)) from a system state \(\sigma_i\) to the succeeding state \(\sigma_{i+1}\) it is required that

\[
\bigwedge_{\omega \in \Omega_i} (\omega_i = \omega) \Rightarrow (\lbrack<\omega_i\rbrack \land \lbrack\geq_{\omega_i}\rbrack \land \lbrack\lbrack_{\omega_i}\rbrack))
\]

holds, where

- \(\lbrack<\omega_i\rbrack\) is a constraint enforcing the preconditions of \(\omega\) for system state \(\sigma_i\),
- \(\lbrack\geq_{\omega_i}\rbrack\) is a constraint enforcing the postconditions of \(\omega\) for system state \(\sigma_{i+1}\), maybe by using \(\sigma_i\) as well, and
- \(\lbrack_{\omega_i}\rbrack\) is a constraint enforcing the frame conditions for the entire transition (i.e., for both system states).

Based on this formulation, afterwards the particular validation or verification objective can be formulated in terms of further specific constraints. For instance, in order to check whether a certain operation \texttt{class::op()} is executable at all, a constraint stating that \(\omega = \omega_1 \lor \ldots \lor \omega_n = \omega_k \cdot \texttt{op()}\) (where \(\omega_1\) to \(\omega_k\) are the possibly instantiated objects of \texttt{class} in \(\sigma_j\) needs to be added. Finally, the complete problem instance is passed to a reasoning engine (solver) which is supposed to determine a satisfying assignment to all variables, i.e., an assignment that satisfies all of the constraints. If the solver returns SAT, i.e., a satisfying assignment has been determined, a corresponding sequence of valid system states and transitions (a so-called witness of the problem instance) can be extracted. Otherwise, if the solver returns UNSAT, it has been proven that no satisfying assignment exists (within the specified problem bounds).\(^5\)

5.2 Employing the Analysis Objective

In the following, we utilize the reasoning scheme reviewed above for the analysis of frame conditions.

\(^3\)In addition to limiting the sequence length, all these approaches require further problem bounds in order to limit the search space, i.e., they need to be provided with a fixed number or at least a range of objects that shall be instantiated as well as a finite domain for all data types.

\(^4\)Note that, in the following, an abstract description is provided which is sufficient for the purposes of this work. For a more detailed treatment of the respective formulation, we refer to (Przigoda et al., 2016b).

\(^5\)Note that the solver will always conclude at some point due to the finite search space.
In order to analyze the consistency for given frame conditions \( F \) regarding an operation \( op \), the formulation simply has to ask “Does there exist a valid execution scenario for operation \( op \)?” \((S^F_{op} \neq \emptyset)\). As validity is ensured implicitly by the general formulation, this boils down to the question whether the operation \( op \) is executable at all. As already discussed above, no further constraints have to be applied to answer this question besides the restriction of \( \omega_1 \) to the operation under consideration.

In order to check whether a pivot scenario \( \omega_p \in P \) given in terms of a pair of a pre- and a poststate is valid \((\omega_p \in S^F_{op})\), the specified values of attributes, links, etc. additionally have to be enforced in the corresponding state.

Example 6. In order to enforce the pivot scenario from Fig. 8(a), the constraints listed in Fig. 12 have to be added.

If these formulations return SAT, it has been shown that \( S^F_{op} \neq \emptyset \) or \( \omega_p \in S^F_{op} \), respectively, and a valid execution scenario can be extracted from the satisfying assignment. If UNSAT is returned, it has been proven that no valid execution scenario exists or that the given scenario \( \omega_p \) is not valid, respectively.

Note that it is possible to let the solver check a set \( P \) of multiple alternative pivot scenarios at the same time. However, in case of SAT, we would not be able to deduce that \( P \) is entirely contained in \( S^F_{op} \) as the found witness only implies that \( P \cap S^F_{op} \neq \emptyset \).

5.2.2 Equivalence

In order to prove the equivalence of two sets of frame conditions \( F_1 \) and \( F_2 \), \((S^F_{op} = S^F_{op})\), we ask the solver to find a counterexample \( \omega \in (S^F_{op} \cap S^F_{op}) \cup (S^F_{op} \setminus S^F_{op}) \), i.e., a scenario that is only valid when enforcing one set of frame conditions, but not the other. Using the standard formulation (cf. Eq. (1)), we can only enforce either \([F_{1,\omega}]\) or \([F_{2,\omega}]\) at the same time. However, as the corresponding constraints are commonly generated in an automatic fashion from the original description of frame conditions, there is no reason why one should not enforce, e.g., \( \lnot[F_{i,\omega}] \) instead of \([F_{i,\omega}] \) \((i = 1, 2)\). Then, only those scenarios would be considered “valid” by the solver which are not compatible with the respective frame conditions. This can be exploited for our purpose by enforcing the constraint

\[
([F_{1,\omega}] \land \lnot[F_{2,\omega}]) \lor ([F_{2,\omega}] \land \lnot[F_{1,\omega}])
\]

instead of \([F_{\omega}]\) in Eq. (1).

If this formulation returns UNSAT, it has been proven that \((S^F_{op} \cap S^F_{op}) \cup (S^F_{op} \setminus S^F_{op}) = \emptyset\). This is logically equivalent to \( S^F_{op} = S^F_{op} \), i.e., both sets of frame conditions are equivalent. If SAT is returned, an execution scenario can be extracted from the satisfying assignment which is valid for exactly one set of frame conditions (but not for the other). This scenario can then be analyzed further.

Note that equivalence can either be checked for the frame conditions of a single operation or for all operations of the considered model at once. However, in the latter case, a possible witness will reveal only one of possibly multiple operations for which the frame conditions are not equivalent.

5.2.3 Independence

Determining dependencies between different sets of frame conditions \( F_1 \) and \( F_2 \) essentially boils down to performing consistency checks on \( F_1 \), \( F_2 \) and \( F_1 \cup F_2 \). Unfortunately, it is in general not possible to automatically derive the constraint \([F_1 \cup F_2,\omega]\) for the frame conditions \( F_1 \cup F_2 \) from the constraints \([F_1,\omega]\) and \([F_2,\omega]\). In fact, only when the frame conditions are specified using modified only statements, the approach presented in (Przigoda et al., 2016a) can be employed to do this automatically. More precisely, the constraints \([F_1,\omega]\) and \([F_2,\omega]\) are constructed using so-called variability maps which store for each model element whether it may be modified or not. Then, the logical disjunction of these maps precisely gives \([F_1 \cup F_2,\omega]\). In all other cases, \( F_1 \cup F_2 \) is required to be specified manually which can be a highly elaborate and non-trivial task.
In order to determine dependencies between changes to model elements from a set \( M = \{m_1, \ldots, m_k\} \) and changes to a model element \( m_0 \notin M \), the solver is asked to determine two different execution scenarios. In these scenarios, all elements from the set \( M \) are required to be modified using constraints like \( \text{modelElement} \neq \text{modelElement}@\text{pre} \) or \( \exists \text{modelElements} \neq \exists (m | m \neq m@\text{pre}) \) (depending on whether a single or multiple instances of the model element are included in the frame), while (1) the model element \( m_0 \) is required to be modified with similar constraints in one scenario and (2) \( m_0 \) is required to keep its value (\( m_0 = m_0@\text{pre} \) or \( m_0 \rightarrow \forall m | m = m@\text{pre} \)) in the other scenario.

If the solver can determine a valid execution scenario in both cases, there is no dependency. If the solver can determine a valid execution scenario only in one case, it follows that \( m_0 \) is either forced to change or to remain constant, respectively. If the solver returns UNSAT in both cases (and the frame conditions are consistent in principle), one can deduce that there already has to be a dependency between the model elements of \( M \) such that not all of them may be changed at once.

Having these problem formulations, the objectives proposed in the previous section can be evaluated automatically using approaches for automated reasoning on UML/OCL models. In the following section, we discuss how the resulting methodology can actually be applied and, beyond that, additionally allows for more elaborated analyses on the considered set of frame conditions.

### 6 Application and Further Potential

We implemented the presented concepts and formulations for the analysis of frame conditions on top of the model verification approach presented in (Przigoda et al., 2016b). Here, the authors propose to translate the verification task into an instance of a Satisfiability Modulo Theories (SMT) problem. The corresponding symbolic formulation is created automatically in terms of the SMT-LIB bit-vector logic QF_BV. Then, the problem instance can be solved using so-called SMT solvers (e.g., Z3 (De Moura and Björner, 2008)). These solvers allow for an efficient traversal of large search spaces and, hence, are suitable to determine precise assignments to the symbolic formulation and, by this, a sequence of transitions satisfying the considered verification objective. A big advantage of this particular approach regarding the analysis of frame conditions is that it natively supports the nothing else changes approach as well as modifies only statements according to the symbolic formulation proposed in (Przigoda et al., 2016a). More precisely, as already indicated above, the constraints \( \exists F \) that enforce a set of frame conditions within the symbolic formulation (cf. Eq. (1)) are realized as variability maps which, for each model element, store whether it may be modified by the corresponding operation call or not. By combining several of these maps for different sets of frame conditions, the required constraints for analyzing equivalence or independence can be generated in a convenient, automatic fashion.

We successfully employed this implementation for the automatic analysis of the objectives introduced above. In fact, the whole analysis presented in Examples 2 (consistency), 3 (equivalence), and 4 (independence) could be performed automatically and the absence or existence of corresponding execution scenarios could be proven formally. This is especially remarkable for the equivalence of the frame conditions provided in Figs. 3, 4 and 6 (cf. Example 3) as the required proof for the absence of a counterexample is very elaborate (if not completely infeasible) to be conducted manually. Using the deductive power and efficiency of established reasoning approaches certainly helped here.

Besides the analysis presented in those examples, the presented methodology offers a large potential for further applications:

- The employed reasoning approach allows to use pivot scenarios that are only partially specified, i.e., values of model elements can be left open and will be assigned by the solver if, and only if, there is a possible assignment that belongs to a valid scenario.

For instance, Figure 13 shows a pivot scenarios where only a few attribute values are actually specified, while the majority is not specified (indicated by a “?”). For the corresponding variables, the solver determines a satisfying assignment (shown in blue color) which extends the partially specified scenario to a completely specified scenario that is valid and compatible with the given frame conditions.

- Frame conditions can be evaluated for aspects like completeness or minimality, i.e., whether they precisely describe the intended frame of change and whether this is done in a somehow optimal fashion.

For instance, if one considers the frame conditions specified in Fig. 4 and drops either of the first two
Figure 13: Partially specified pivot scenario.

modifies only statements (Line 4 or 5), one obtains frame conditions that are incompatible with any scenario in which the performed checks succeed and access is granted. Dropping the third statement (Line 6) makes it impossible to store the card’s ID in case of success. Overall, this shows that the initial set of frame conditions is already minimal. A similar methodology using consistency and equivalence checks can be applied on any model.

- The methods can be used to evaluate the proposals for frame conditions that are automatically generated from a given model using solutions as proposed in (Niemann et al., 2015b).

For instance, for the operation goThrough() the approach from (Niemann et al., 2015b) suggests to consider the model elements self.greenLightOn and building.inside as affected (with high probability) and to have a more thorough look at self.entry, self.currentlyAuthorized, and self.redLightOn (which also occur in the postconditions or may have a dependency via invariants, respectively).

Including all mentioned model elements in the frame of change (e.g., using corresponding modifies only statements) yields consistent frame conditions, but allows for much more changes than intended by the designer. Consequently, the impact of the individual statements, e.g., on the validity of pivot scenarios, needs to be analyzed and unnecessary statements have to be dropped.

Overall, the proposed method allows for an efficient, automatic analysis of frame conditions with respect to the three primary analysis objectives of consistency, equivalence and independence, and also provides potential for a variety of further applications beyond that.

7 Conclusions

In this work, we considered the analysis of frame conditions in UML/OCL models. While several proposals and formalisms for specifying frame conditions exist, it remains non-trivial to define them properly. In fact, no corresponding methods or tools have been developed so far which can guarantee that the derived frame conditions indeed complete the model description in the intended way. We addressed this gap by proposing a set of analysis objectives (consistency, equivalence, and independence) together with a formulation that allows for performing corresponding analyses using automated reasoning engines. Moreover, we implemented the proposed concepts on top of an established approach for model validation and verification. By this, a method and also a corresponding tool becomes available that allows for the dedicated analysis of frame conditions with a similar performance as many established approaches for the validation and verification of UML/OCL models in general. More precisely, the method benefits from the same deductive power of automatic reasoning engines as well as the same efficiency and scalability, but now additionally targets frame conditions rather than pure UML/OCL descriptions only.

Acknowledgments

This work was supported by the German Federal Ministry of Education and Research (BMBF) within the project SELFIE under grant no. 01IW16001 and the German Research Foundation (DFG) within the Reinhart Koselleck project under grant no. DR287/23-1.

REFERENCES


